Review Problems 1

1) Say everything you can about the following matrix.

$$\begin{bmatrix} 5 & 10 & 0 & 0 & 0 \\ 1 & 2 & 7 & 7 & 7 \\ 0 & 1 & 7 & 7 & 7 \\ 0 & 0 & 3 & 3 & 3 \end{bmatrix}$$

In particular, address the following incomplete list of interesting things:

- What is its echelon form? Identify the elementary row operations you used.
- What is its reduced echelon form?
- Are the columns linearly independent or dependent?
- What is the span of the columns?
- If linearly independent, why? If linearly dependent, illustrate the linear combination that is zero.
- Construct a homogenous matrix equation. Which are the free and leading variables? How many solutions are there?
- Construct a nonhomogenous matrix equation. How many solutions are there?
- Solve your nonhomogenous equation, what is the general solution?
- What is the associated linear transformation?
- What is the domain and range of the associated linear transformation?
- Is the linear transformation one-to-one? Onto?

2) Say everything you can about the following vectors.

$$\begin{bmatrix} 3 \\ 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

In particular, all the previous items previous problem applies to this as well.

3) How about these vectors?

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

4) Find linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^4$ that is one-to-one but not onto.

5) Find a system of equations in which the solutions form a "$\mathbb{R}^2$ space".

6) Find a 3×3 matrix whose inverse requires exactly two elementary row operations to find.